SAGA–HE–103
June 10, 1996

Explicit CP Breaking and Electroweak Baryogenesis

Koichi Funakubo[1], Akira Kakuto[*2],
Shoichiro Otsuki[*3] and Fumihiko Toyoda[*4]

Department of Physics, Saga University, Saga 840 Japan
[*]Department of Liberal Arts, Kinki University in Kyushu, Iizuka 820 Japan

Abstract

We investigate spatial behaviors of the CP-violating angle θ by solving the equation of motion of the two-Higgs-doublet model in the presence of a small explicit CP breaking δ. The moduli of the two Higgs scalars are fixed to be the kink shape with a common width. In addition to solutions $\theta \sim O(\delta)$ in all the region, we find several sets of two solutions of opposite signs, whose magnitudes become as large as $O(1)$ around the surface of the bubble wall while the CP violation in the broken phase limit is of $O(\delta)$. Such set of solutions not only yield sufficient amount of the chiral charge flux, but also avoid the cancellation in the net baryon number because of a large discrepancy in their energy densities driven nonperturbatively by the small δ.

[1]e-mail: funakubo@cc.saga-u.ac.jp
[2]e-mail: kakuto@fuk.kindai.ac.jp
[3]e-mail: otks1scp@mbox.nc.kyushu-u.ac.jp
[4]e-mail: ftoyoda@fuk.kindai.ac.jp

1 Introduction

The electroweak baryogenesis[1] strongly depends on the CP-violating angle $\theta(z)$ created at the first-order phase transition via bubble nucleation, where z is the spatial coordinate perpendicular to the 'planar' bubble wall. As is well known, one of difficulties of the electroweak baryogenesis in the minimal standard model may be that the explicit CP-breaking parameters of the Kobayashi-Maskawa scheme are too small to generate the observed baryon asymmetry of the universe.

In a previous article [2][1], we examined the behaviors of $\theta(z)$ that is spontaneously generated in the two-Higgs-doublet model, by assuming that the moduli of the two neutral scalars, $\rho_i(z)(i = 1, 2)$, have the kink shape of a common width. One of the solutions we presented has a remarkable spatial dependence such that $\theta(z)$ becomes as large as $O(1)$ around the surface of the bubble wall while it completely vanishes deep in the broken and symmetric phase regions. Such a solution may be of much importance in the electroweak baryogenesis since, as we showed there, it does yield a large amount of the chiral charge flux through the wall surface at the phase transition.

In this article we examine $\theta(z)$ in the two-Higgs-doublet model in the presence of an explicit CP breaking δ at the transition temperature T_C, also by fixing the moduli to be the common kink shape. The magnitude of the breaking parameter δ may not be largely different from those of the Kobayashi-Maskawa scheme even when finite-temperature corrections are taken into account. Then a naive guess would be that $\theta(z)$ remains of $O(\delta)$ in all the region between the broken and symmetric phase limits. Actually we give such solutions obtained analytically. On the other hand, we find several solutions whose $\theta(z)$'s become as large as $O(1)$ around the wall surface, as important as the one in the case of spontaneous CP violation($\delta = 0$).

As pointed out by Comelli *et al.*[3], the explicit CP breaking may be necessary to avoid the complete cancellation in the net baryon number expected from the symmetry of the solution $\theta(z) \longleftrightarrow -\theta(z)$ in the case of $\delta = 0$. In the presence of $\delta \neq 0$, the energy density of the bubble with $\theta^+(z)$ close to $\theta^{\delta=0}(z)$ and that with $\theta^-(z)$ close to $-\theta^{\delta=0}(z)$ no more degenerate. We give an estimate that the relative enhancement factor due to the energy difference between the two kinds of bubbles could be as large as $O(10)$ even for $\delta \sim O(10^{-3})$. Such a large relative enhancement factor would favor the formation of one of the two kinds of bubbles and guarantee the baryon asymmetry of the universe.

In Section 2 we introduce the breaking parameter δ into the standard two-Higgs-doublet potential, and give the equation of motion for $\theta(z)$. In Section 3, we discuss the

[1] Ref.[2] is referred to as I.

boundary conditions to be satisfied by $\theta(z)$. In Section 4 we show examples of $\theta(z) \sim O(\delta)$ obtained analytically. Several solutions of $\theta(z) \sim O(1)$ around the bubble wall are presented in Section 5. In Section 6, the energy difference and the relative enhancement factor are estimated. Section 7 is devoted to concluding remarks.

2 Explicit CP Breaking and Equation for θ

In order to clarify essential roles played by δ, we examine the problem under the following simplified conditions:

(1) One breaking parameter δ is introduced into the standard two-Higgs-doublet potential as $(m_3^2 e^{-i\delta}\Phi_1^\dagger\Phi_2 + h.c.)$ while $\lambda_5, \lambda_6, \lambda_7 \in \mathbf{R}$ is assumed.

(2) The magnitude of δ at T_C is small enough.

(3) Let VEV's of the respective neutral components of $\Phi_i (i = 1, 2)$ be $(1/\sqrt{2})\rho_i(z)e^{i\theta_i(z)}$. The two moduli $\rho_i(z)$'s are assumed to take the kink shape of a common width $1/a$:

$$\rho_1(z) = v\cos\beta(1 + \tanh(az))/2, \quad \rho_2(z) = v\sin\beta(1 + \tanh(az))/2, \tag{2.1}$$

where $v\cos\beta$ and $v\sin\beta$ are VEV's of Φ_1 and Φ_2 respectively in the broken phase limit.

Here the same convention as I is used for the parameters in the effective potential. The condition (1) may not be specific since δ is induced from the soft-SUSY-breaking parameters in the MSSM[3].

Following I, we postulate the effective potential V_{eff}, which is considered to include the radiative and finite-temperature corrections, as follows:

$$\begin{aligned} V_{eff}(\rho_1, \rho_2, \theta) &= \frac{1}{2}m_1^2\rho_1^2 + \frac{1}{2}m_2^2\rho_2^2 + m_3^2\rho_1\rho_2\cos(\delta + \theta) + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\ &+ \frac{\lambda_3 - \lambda_4}{4}\rho_1^2\rho_2^2 - \frac{\lambda_5}{4}\rho_1^2\rho_2^2\cos 2\theta - \frac{1}{2}(\lambda_6\rho_1^2 + \lambda_7\rho_2^2)\rho_1\rho_2\cos\theta \\ &- \left(A\rho_1^3 + B\rho_1^2\rho_2\cos\theta + C\rho_1\rho_2^2\cos\theta + D\rho_2^3\right), \end{aligned} \tag{2.2}$$

where $\theta \equiv \theta_1 - \theta_2$. Here the ρ^3 terms just above are expected to arise at finite temperatures so that the kink shape moduli (2.1) of the bubble wall are realized for $\theta(z) = 0$ and $\delta = 0$ at T_C. Then several relations among the parameters in (2.2) are required as given in I.

In terms of dimensionless coordinate $y \equiv (1 - \tanh(az))/2$, the equation of motion for $\theta(y)$ derived from $V_{eff}(\rho_1 = v\cos\beta(1 - y), \rho_2 = v\sin\beta(1 - y), \theta(y))$ is:

$$\begin{aligned} & y^2(1-y)^2\frac{d^2\theta(y)}{dy^2} + y(1-y)(1-4y)\frac{d\theta(y)}{dy} \\ = \; & b\sin(\delta + \theta(y)) + [c(1-y)^2 - e(1-y)]\sin\theta(y) + \frac{d}{2}(1-y)^2\sin(2\theta(y)), \end{aligned} \tag{2.3}$$

where

$$
\begin{aligned}
b &\equiv -\frac{m_3^2}{4a^2 \sin\beta\cos\beta}, \\
c &\equiv \frac{v^2}{32a^2}(\lambda_1 \cot^2\beta + \lambda_2 \tan^2\beta + 2(\lambda_3 - \lambda_4 - \lambda_5)) - \frac{1}{2\sin^2\beta\cos^2\beta} \\
&= \frac{v^2}{8a^2}(\lambda_6 \cot\beta + \lambda_7 \tan\beta), \\
d &\equiv \frac{\lambda_5 v^2}{4a^2}, \\
e &\equiv \frac{v}{4a^2\sin^2\beta\cos^2\beta}\left(A\cos^3\beta + D\sin^3\beta - \frac{4a^2}{v}\right) \\
&= -\frac{v}{4a^2}\left(\frac{B}{\sin\beta} + \frac{C}{\cos\beta}\right).
\end{aligned}
\tag{2.4}
$$

In addition, the requirement that $(\rho_1, \rho_2) = (0, 0)$ and $(\rho_1, \rho_2) = (v\cos\beta, v\sin\beta)$ to be local minima of V_{eff} with $\theta = 0$ leads to inequalities among the parameters for $\delta = 0$:

$$
b > -1, \quad b - 2e + 3c > -1 + (\lambda_3 - \lambda_4 - \lambda_5)v^2/4a^2. \tag{2.5}
$$

Note that the explicit CP violation $\delta \neq 0$ breaks the symmetry $\theta(y) \longleftrightarrow -\theta(y)$ of (2.3), which is allowed in the case of $\delta = 0$.

3 Boundary Conditions Satisfied by θ

Broken phase limit

Suppose that, at $y \sim 0$, $\theta(y)$ is given as

$$
\theta(y) = \theta_0 + a_0 y^\nu + (\text{h.o.t.}(y)) \qquad (\nu > 0, a_0 \neq 0), \tag{3.1}
$$

where (h.o.t.(y)) means (higher order terms of y). Inserting this into (2.3), we have

$$
\begin{aligned}
y^\nu[\nu^2 a_0 + (\text{h.o.t.}(y))] &= [W_0 + W_1 y + W_2 y^2] \\
&+ y^\nu[W_3 a_0 + (\text{h.o.t.}(y))] \\
&+ y^{2\nu}[(-a_0^2/2!)W_4 + (\text{h.o.t.}(y))],
\end{aligned}
\tag{3.2}
$$

where the y^ν terms on the right hand side come from $\sin(\theta(y) - \theta_0)$ and the $y^{2\nu}$ terms from $\cos(\theta(y) - \theta_0) - 1$.

That $\nu > 0$ requires

$$
W_0 \equiv b\sin(\delta + \theta_0) + (c - e + d\cos\theta_0)\sin\theta_0 = 0, \tag{3.3}
$$

from which

$$\tan\theta_0 = -\frac{b\sin\delta}{b\cos\delta + c - e + d\cos\theta_0}. \tag{3.4}$$

Without loss of generality, let us restrict θ_0 as $-\pi/2 \le \theta_0 < 3\pi/2$. Because of $b \propto m_3^2 \ne 0$ necessary to introduce δ, $\theta_0 \ne 0, \pi$. Making use of (3.3)

$$\begin{aligned}
W_1 &\equiv (-2c + e - 2d\cos\theta_0)\sin\theta_0 = 2b\sin(\delta+\theta_0) - e\sin\theta_0, && (3.5)\\
W_2 &\equiv (c + d\cos\theta_0)\sin\theta_0 = -b\sin(\delta+\theta_0) + e\sin\theta_0, && (3.6)\\
W_3 &\equiv b\cos(\delta+\theta_0) + (c-e)\cos\theta_0 + d\cos(2\theta_0) \\
&= -d\sin^2\theta_0 - b(\sin\delta/\sin\theta_0), && (3.7)\\
W_4 &\equiv b\sin(\delta+\theta_0) + (c-e)\sin\theta_0 + 4d\sin\theta_0\cos\theta_0 \\
&= 3d\sin\theta_0\cos\theta_0. && (3.8)
\end{aligned}$$

If we take $\nu = 1$ for illustration, a_0 is determined as

$$a_0 = \frac{2b\sin(\delta+\theta_0) - e\sin\theta_0}{1 + d\sin^2\theta_0 + b(\sin\delta/\sin\theta_0)} \tag{3.9}$$

from $\nu^2 a_0 = W_1 + W_3 a_0$. When ν is not an integer, a_0 is not determined from the lower order relations.

Symmetric phase limit

Suppose that, at $\zeta \equiv 1 - y \sim 0$, $\theta(\zeta)$ is given as

$$\theta(\zeta) = \theta_1 + b_0\zeta^\mu + (\text{h.o.t.}(\zeta)) \qquad (\mu > 0, b_0 \ne 0). \tag{3.10}$$

Inserting this into (2.3), we have

$$\begin{aligned}
\zeta^\mu[\mu(\mu+2)b_0 + (\text{h.o.t.}(\zeta))] &= [U_0 + U_1\zeta + U_2\zeta^2] \\
&+ \zeta^\mu[U_3 b_0 + (\text{h.o.t.}(\zeta))] && (3.11)\\
&+ \zeta^{2\mu}[(-b_0^2/2!)U_4 + (\text{h.o.t.}(\zeta))].
\end{aligned}$$

That $\mu > 0$ requires $U_0 \equiv b\sin(\delta+\theta_1) = 0$, so that

$$\theta_1 = \ell\pi - \delta \qquad (\ell = 0, \pm 1, \pm 2, \cdots). \tag{3.12}$$

Making use of this,

$$\begin{aligned}
U_1 &\equiv -e\sin\theta_1 = (-1)^\ell e\sin\delta, && (3.13)\\
U_2 &\equiv (c + d\cos\theta_1)\sin\theta_1 = -((-1)^\ell c + d\cos\delta)\sin\delta, && (3.14)\\
U_3 &\equiv b\cos(\delta+\theta_1) = (-1)^\ell b, && (3.15)\\
U_4 &\equiv U_0 = 0. && (3.16)
\end{aligned}$$

If we take $\mu = 1$ for illustration, b_0 is given from $\mu(\mu+2)b_0 = U_1 + U_3 b_0$ as follows:
(i) For $e \neq 0$ and $b \neq (-1)^\ell 3$,

$$b_0 = \frac{(-1)^\ell e \sin\delta}{3 - (-1)^\ell b}. \tag{3.17}$$

(ii) For $e = 0$, $b = (-1)^\ell 3$ while b_0 is not specified *a priori*.

Our task is to solve the nonlinear and inhomogeneous differential equation (2.3) with the boundary conditions of θ_0 and θ_1 (the two-point boundary value problem). We take the simplest boundary conditions that $\theta_0 \sim O(\delta)$ and $\theta_1 = -\delta$ throughout, which will give the lowest energy configurations.

4 Examples of Solutions $\theta \sim O(\delta)$

Assuming that $\theta(y) \sim O(\delta)$ in the interval $y \in [0,1]$, let us linearize (2.3) as [4]

$$\begin{aligned} & y^2(1-y)^2 \frac{d^2\theta(y)}{dy^2} + y(1-y)(1-4y)\frac{d\theta(y)}{dy} \\ = & \Big[b + (c+d)(1-y)^2 - e(1-y)\Big]\theta(y) + b\delta. \end{aligned} \tag{4.1}$$

Taking $\nu = 1$ for illustration, suppose, for a moment, to search for the solution as the initial value problem: $\theta(y)$ starting from $\theta(0) = \theta_0$ with $\theta'(0) = a_0$ has to hit against $\theta(1) = \theta_1$. A naive guess would be that the solution $\theta(y)$ should be of $O(\delta)$, since θ_0 by (3.4), a_0 by (3.9) and $\theta_1 = -\delta$ are all of $O(\delta)$ in general. Actually, numbers of such solutions to the linearized equation are obtained even analytically. We show a few examples.

Example 1. We assume $\theta(y) = p_0 + p_1 y$. Given (b,d), a set of algebraic linear equations for (p_0, p_1) obtained from (4.1) are satisfied for $(c = 4-d, e = (7 \pm \sqrt{1+16b^2})/2)$, giving $p_0 = -(b/(b+c+d-e)) \times \delta$ and $p_1 = -(4/(e-3))p_0$. For this $\theta(y)$ to satisfy the boundary condition $\theta_1 = -\delta$, the unique choice is $(b,e) = (3,0)$. Namely, for $(b,c,d,e) = (3, 4-d, d, 0)$, we have a linear solution

$$\theta(y) = -((3+4y)/7) \times \delta. \tag{4.2}$$

Example 2. In the similar way, for $(b,c,d,e) = (3, 10-d, d, 0)$, we have a quadratic solution

$$\theta(y) = -((3+5y+5y^2)/13) \times \delta. \tag{4.3}$$

In the case of spontaneous CP violation, we obtained an almost linear solution $\theta^{\delta=0}(y)$ shown in Fig.1 of I, for which $-\theta^{\delta=0}(y)$ is also a solution. However, once $\delta \neq 0$ is given, $-\theta(y)$ of (4.2) or (4.3) is no more a solution. Although we have no cancellation in the net baryon number in this case, such solutions of $\theta(y) \sim O(\delta)$ would contribute at most marginally to the baryon asymmetry[4].

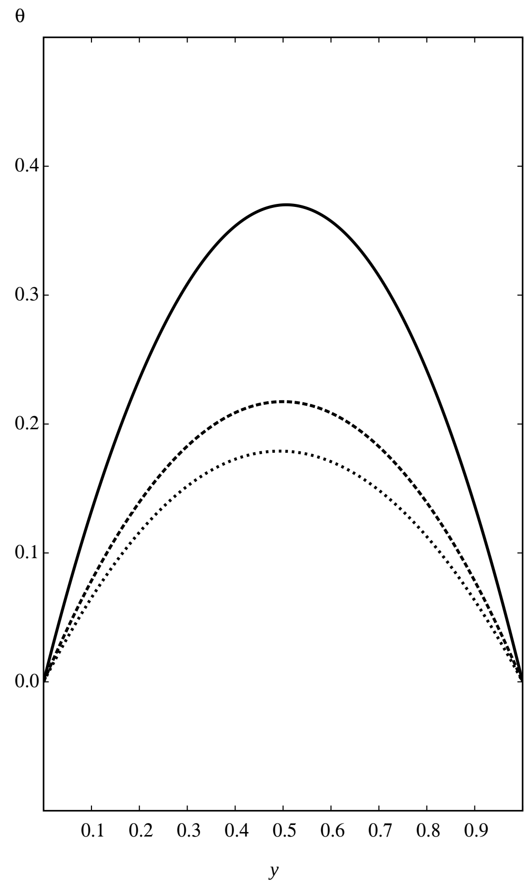

Figure 1: Numerical solutions of $\theta^{\delta=0}(y)$ for $(b^{(0)}, c^{(0)}, d^{(0)}, e^{(0)}) = (3, 12.2, -2, 12.2)$ (solid curve), $(3, 8.98, 1, 11.98)$ (dashed curve) and $(3, 10, -0.2, 11, 8)$ (dotted curve). The first is the one given in Fig.3 of I. For these, $\theta_0 = \theta_1 = 0$.

5 Solutions $\theta \sim O(1)$ around Bubble Wall

Suppose that for $\delta = 0$ we have a set of solutions $\pm\theta^{\delta=0}(y)$ which are not $O(\delta)$ but $O(1)$. For $|\delta|$ small enough, we could expect a solution $\theta^{+}(y)$ close to $\theta^{\delta=0}(y)$ and another one $\theta^{-}(y)$ close to $-\theta^{\delta=0}(y)$. The both $\theta^{\pm}(y)$ satisfy the same boundary conditions $\theta_0 \sim O(\delta)$ and $\theta_1 = -\delta$ but $\theta^{-}(y) \neq -\theta^{+}(y)$. Before giving numerical solutions $\theta^{\pm}(y)$, we show how such solutions of $O(1)$ are obtained from $\pm\theta^{\delta=0}(y)$ for sufficiently small $|\delta|$. For definiteness, we take $\nu = \mu = 1$.

Solutions $\pm\theta^{\delta=0}(y)$

In Fig.1 we show three $\theta^{\delta=0}(y)$ satisfying the boundary conditions $\theta_0 = \theta_1 = 0$ together with the corresponding parameters $(b^{(0)}, c^{(0)}, d^{(0)}, e^{(0)})$. One of them is the one given in Fig.3 of I. Because $W_1 = U_1 = 0$ for $\delta = 0$, the parameters are required to satisfy

$$b^{(0)} + c^{(0)} - e^{(0)} + d^{(0)} = \nu^2 = 1 \tag{5.1}$$

from $(\nu^2 - W_3|_{\delta=0}) a_0^{\delta=0} = 0$, and

$$b^{(0)} = \mu(\mu+2) = 3 \tag{5.2}$$

from $(\mu(\mu+2) - U_3|_{\delta=0}) b_0^{\delta=0} = 0$. Of course, once we get a solution $\theta^{\delta=0}(y)$, its $a_0^{\delta=0}$ and $b_0^{\delta=0}$ have been uniquely determined to match the boundary conditions. As is clear from the figure, $\theta'(0) = a_0^{\delta=0} \sim O(1)$ and $\theta'(1) = -b_0^{\delta=0} \sim O(1)$. These are what enable $\theta(y \simeq 0.5)$ to become as large as $O(1)$.

Solutions $\theta^{\pm}(y)$

This suggests that we could have solutions $\theta^{\pm}(y) \sim O(1)$ for $\delta \neq 0$ if $a_0 \sim b_0 \sim O(1)$. As shown in Section 3, they are determined from the lower order relations when $\nu = \mu = 1$. Now we seek a set of parameters (b, c, d, e), which incorporates such solutions for $\nu = \mu = 1$.[2]

Put $b = b^{(0)} + \Delta b \times \delta$, $c = c^{(0)} + \Delta c \times \delta$, $d = d^{(0)}$ and $e = e^{(0)} + \Delta e \times \delta$. For sufficiently small $|\delta|$, we have from (3.4), with the use of (5.1),

$$\theta_0 \sim -\frac{b}{b+c-e+d} \times \delta \sim -b \times \delta, \quad i.e., \quad \delta/\theta_0 \sim -1/b. \tag{5.3}$$

This implies that the denominator of a_0 in (3.9) is not of $O(1)$ but $O(\delta)$. Since the numerator of a_0 is $O(\delta)$, a_0 is now a quantity of $O(1)$: $a_0 \sim b^{(0)}(2c^{(0)} - e^{(0)} + 2d^{(0)})/(-\Delta b - \Delta c + \Delta e)$. Because of (5.2), b_0 in (3.17) is also a quantity of $O(1)$ for $e^{(0)} \neq 0$: $b_0 \sim -e^{(0)}/\Delta b$. By suitably adjusting $(\Delta b, \Delta c, \Delta e)$ in such a way that $a_0 \sim a_0^{\delta=0} \sim O(1)$ and $b_0 \sim b_0^{\delta=0} \sim O(1)$ match the boundary conditions, we obtain a desired solution $\theta^+(y)$ close to $\theta^{\delta=0}(y)$. For the same $(\Delta b, \Delta c, \Delta e)$ or (b, c, d, e), we can find another desired solution $\theta^-(y)$ close to $-\theta^{\delta=0}(y)$. In Fig.2 we show an example of $\theta^{\pm}(y)$. Note that $-\theta^-(y)$ and $\theta^+(y)$ do not coincide with but considerably differ from each other. Note also that, while $\theta_0 \sim -b \times \delta$ and $\theta_1 = -\delta$, $\theta^{\pm}(y)$ deviate nonperturbatively from the corresponding $\pm\theta^{\delta=0}(y)$ in the intermediate region. Presumably this may be due to the nonlinearity and the singular effects for $\theta''(y)$ near $y = 0, 1$ of the differential equation (2.3).

We can also find several solutions for other sets of the parameters (b, c, d, e), as long as they do not change the global structure of the effective potential. Of course, they would not have $\nu = \mu = 1$ in general. The numerical method is based on the relaxation algorithm. For sufficiently small δ, say $\delta \in (0, 0.001]$, the both types of solutions $\theta^{\pm}(y)$ can be found by starting from initial configurations with opposite signs, respectively. As we increase δ, only $\theta^-(y)$ can be obtained starting from *any* initial configuration. This would

[2] In practice, we are given with the effective potential, so that (b, c, d, e) are fixed. Then ν and μ are determined by solving the equation for $\theta(y)$. Since our purpose here is to show the possibility to have such solutions, we trace an unusual course to find them.

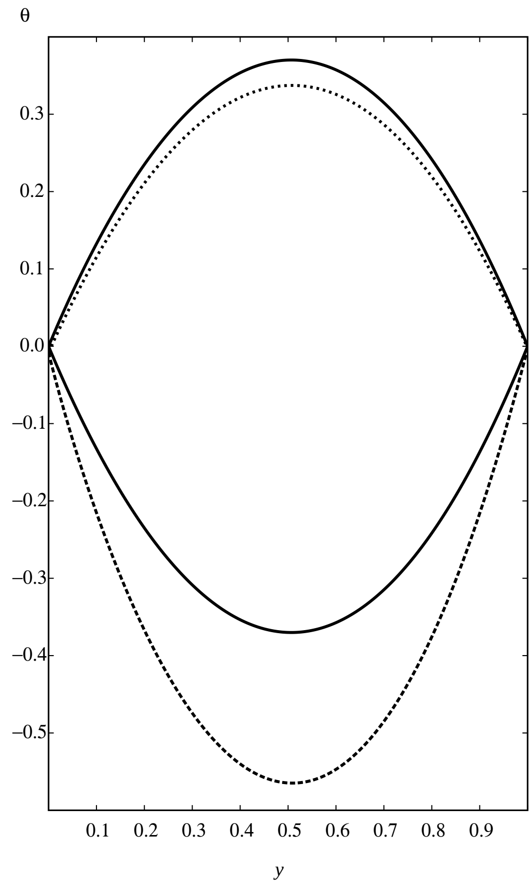

Figure 2: Numerical solutions of $\theta^{\pm}(y)$ in which $\delta > 0$ is incorporated into the symmetric pairs of the solid-curve solution $\theta^{\delta=0}(y)$ for $(b^{(0)}, c^{(0)}, d^{(0)}, e^{(0)}) = (3.12.2, -2, 12.2)$ in Fig. 1 by the prescription explained in the text. These pairs are given respectively by the upper and lower solid curves. For $\delta = 0.0025$ and $(b, c, d, e) = (2.98005, 12.178375, -2, 12.2)$, $\theta^{-}(y)$ is given by the dashed curve and $\theta^{+}(y)$ by the dotted one. They have the common boundary values: $\theta_0^{\pm} = -3.109066 \times \delta \sim -b^{(0)} \times \delta$ and $\theta_1 = -\delta$.

mean that for larger δ, $\theta^+(y)$ becomes more unstable or even does not exist [3]. Obviously the critical value of δ, above which $\theta^+(y)$ is not found, depends on the parameters in the potential (b, c, d, e). Although its value is about $O(10^{-3}) \sim O(10^{-2})$ in all the cases we studied, we could not determine its definite value, since it might depend on details in the algorithm, such as the convergence parameter or initial configurations.

6 Energy Density and Enhancement Factor

The energy density of the wall per unit area is given by

$$\begin{aligned}
\mathcal{E} &= \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2} \sum_{i=1,2} \left[\left(\frac{d\rho_i}{dz} \right)^2 + \rho_i^2 \left(\frac{d\theta_i}{dz} \right)^2 \right] + V_{eff}(\rho_1, \rho_2, \theta) \right\} \quad (6.1) \\
&= \int_0^1 dy \left\{ ay(1-y) \sum_{i=1,2} \left[\left(\frac{d\rho_i}{dy} \right)^2 + \rho_i^2 \left(\frac{d\theta_i}{dy} \right)^2 \right] + \frac{1}{2ay(1-y)} V_{eff}(\rho_1, \rho_2, \theta) \right\}, \\
&\equiv av^2/3 + \mathcal{E}[\theta], \quad (6.2)
\end{aligned}$$

where the first term of the above line is the energy density of the trivial solution $\theta^{\delta=0}(y) = 0$. The second term is contributed from a path $\theta(y)$ that connects the broken and symmetric vacua. For the trivial solution this term vanishes.

$\mathcal{E}$ has two degenerate minima corresponding to $\pm\theta^{\delta=0}(y) \neq 0$. For the solid-curve solutions in Figs.1 and 2, we have

$$\mathcal{E}[\pm\theta^{\delta=0}] = -2.056 \times 10^{-3} av^2 \sin^2\beta \cos^2\beta. \quad (6.3)$$

That $\delta \neq 0$ breaks the degeneracy. For $\theta^{\pm}(y)$ shown in 2, in which $\delta = 0.0025$ is incorporated into these $\pm\theta^{\delta=0}(y)$, we actually find that $0 > \mathcal{E}[\theta^+] > \mathcal{E}[\pm\theta^{\delta=0}] > \mathcal{E}[\theta^-]$, the energy difference being

$$\Delta\mathcal{E} \equiv \mathcal{E}[\theta^-] - \mathcal{E}[\theta^+] = -1.917 \times 10^{-2} av^2 \sin^2\beta \cos^2\beta. \quad (6.4)$$

This negative $\Delta\mathcal{E}$ means that the formation of the bubble with $\theta^-(y)$ is favored over that with $\theta^+(y)$. The relative enhancement factor is given by

$$\exp\left(-\frac{4\pi R_C^2 \Delta\mathcal{E}}{T_C} \right), \quad (6.5)$$

where the radius of the critical bubble R_C is approximately given by $\sqrt{3F_C/(4\pi a v^2)}$ with F_C being the free energy of the critical bubble. Various authors estimate $F_C \sim (145 \sim$

[3]This disparity in $\theta^{\pm}(y)$ would be triggered by $\theta_0, \theta_1 < 0$ for $\delta > 0$ in our choice and amplified nonperturbatively as remarked above.

160)T. If we take $F_C = 145T$ and $\tan\beta = 1$, the enhancement factor is

$$\exp\left(-\frac{4\pi R_C^2 \Delta\mathcal{E}}{T_C}\right) = 8.05. \tag{6.6}$$

Such a large relative enhancement factor of $O(10)$ between the bubble with $\theta^-(y)$ and that with $\theta^+(y)$ would surely avoid the cancellation in the chiral charge flux and guarantee the survival of the net baryon number of the universe.

7 Concluding Remarks

Of course, the simplified condition (3) at the beginning of Section 2 that the wall moduli are fixed to be the kink shape is invalid for the solutions $\theta(y) \sim O(1)$, and we have to solve a set of coupled equations for $\theta(y)$ and $\rho_i(y)$ as done in [5]. As shown there, $\theta(y)$ remains to be $O(1)$ though its form is modified to certain extent while $\rho_i(y)$'s largely deviate from the kink shape.[4] The kink-shape approximation is valid only for the solutions of $O(\delta)$ as given in Section 4. Though they have no counter partners as remarked there, the net baryon number would remain at most marginally after the completion of the phase transition.

On the other hand, as we gave an estimate in I for the solid-curve solution $\theta^{\delta=0}(y)$ in Fig.1, such $\theta^{\pm}(y)$ that is able to become $O(1)$ around the bubble wall could supply an efficient chiral charge flux through the wall surface at the phase transition. Note that in the broken phase limit the CP violation is given by $\sin(\delta + \theta_0) \sim \delta$ at T_C, so that, if $|\delta|$ is small enough, there should be no contradictions with the present experimental bounds. Of course the CP violation completely vanishes in the symmetric phase limit because of $\rho_i = 0$ there. That solutions with such features as presented here are allowed in a realistic model would be highly significant in any scenario of the electroweak baryogenesis, since there may be numbers of possible mechanisms to diminish the net chiral charge or the net baryon number before the completion of the phase transition.

Our estimate in Section 6 suggests an interesting possibility that the effect of a small $|\delta|$ is nonperturbatively amplified to yield a large relative enhancement factor to favor the formation of only one of the two kinds of the bubbles, which are the symmetric partners in the absence of the explicit CP violation. Further our numerical analysis suggests that the bubble with higher energy would be metastable or could not exist for larger $|\delta|$. If the possibility is realized, we could be free from the disgusting cancellation in the baryon number of the universe.

[4] For a numerical method how to obtain the chiral transmission and reflection coefficients in such cases, see [6].

This work was partially supported by Grant-in-Aid for Encouragement of Young Scientist of the Ministry of Education, Science and Culture, No.08740213 (K.F.).

References

[1] For a review see, A. Cohen, D. Kaplan and A. Nelson, Ann. Rev. Nucl. Part. Sci. **43** (1993) 27.

[2] K. Funakubo, A. Kakuto, S. Otsuki, K. Takenaga and F. Toyoda, Prog. Theor. Phys. **94** (1995) 845.

[3] D. Comelli, M. Pietroni and A. Riotto, Nucl. Phys. **B412** (1994) 441.

[4] J. M. Cline, K. Kainulainen and A. P. Vischer, hep-ph/9506284 (1995).

[5] K. Funakubo, A. Kakuto, S. Otsuki, and F. Toyoda, in preparation.

[6] K. Funakubo, A. Kakuto, S. Otsuki, and F. Toyoda, Prog. Theor. Phys. **95** (1996) 929.